Policy Distortions and Aggregate Productivity: The Role of Idiosyncratic Shocks

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Abstract

We study the implications of idiosyncratic shocks on the effects of resource misallocation across heterogeneous plants. We consider that plant productivity follows a Brownian process and we use the forward Kolmogorov equation to analytically characterize the endogenous distribution of entering plants. We show that when plant’s operational profits are non constant, exit decision is equivalent to asking whether the plant option value is non-negative. The model reproduces the fact that cross-sectional dispersion in productivity is positively correlated with time-series volatility of productivity. Our main contribution is to show that if one is calibrating/estimating a model without shocks and endogenous entry, to fit data generated from a model with shocks and endogenous entry, one will underestimate fixed cost of production and overestimate TFP distortions.

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1 Introduction

Restuccia and Rogerson (2008) show that policy distortions faced by individual plants can lead to decreases in output and in TFP in the range of 30 to 50 percent. Similarly, Guner et al (2008) find that size dependent policies decrease output per plant and plant size by 25.6 and 20 percent respectively. After these findings, an increasing interest on the effects of resource misallocation across heterogeneous plants has emerged. Hsieh and Klenow (2009) find large deviations in the resource allocation across firms of India and China that explains differences in TFP relative to U.S. in the range of 40 to 60 percent. Similar results are obtained by Neumeyer and Andleris (2010) for the Argentine manufacturing sector.

Much of this literature is concerned with understanding the role of distortions on entry decisions for a given distribution of plants. In order to simplify the problem, this literature assumes that productivity for a given plant is constant over time. This assumption has two important implications. On the one hand, under this assumption plants have constant operational profits and therefore, once they enter the industry, they have no incentive to exit it. On the other hand, entering plants are the only ones deciding whether to exit the industry or not, that is to say, entering plants decide to remain when operational profits are non-negative. If the latter condition is met, no further exit decisions need to be taken.

In this paper we consider policy distortions in a model where plants face idiosyncratic shocks, as in Hopenhayn (1992) and Hopenhayn and Rogerson (1993). As Restuccia and Rogerson (2008) we also consider that low productivity plants receive subsidies and high productivity plants pay taxes. Finally, as Luttmer (2007), we consider that plant productivity follows a Brownian process and we use the forward Kolmogorov equation to analytically characterize the endogenous distribution of entering plants.

What do we learn from our model? Introducing idiosyncratic shocks in the model implies that plants have non-constant operational profits and as a result there is an endogenous exit margin. On the one hand, incumbent plants must decide each period whether to remain or not in the industry. The exit decision is
now equivalent to asking whether the plant option value is non-negative. Therefore, plants with non-positive profits may remain in the industry. On the other hand, if there are shocks and endogenous exit, plants are more eager to enter since they are not stuck with a particular productivity level. Hence, compared to a world without shocks, there will be low productivity plants that are active (waiting for better days). As a result the productivity of the marginal entering plant and TFP will decrease.

Moreover, we analytically characterize cross-sectional dispersion in productivity to show that if idiosyncratic shocks are more time-series volatile, plant’s option value increases and cross-sectional dispersion in productivity increases. This is consistent with Collard-Wexler et al. (2011) findings. Using data from the World Bank’s Enterprise Data, on 5,010 establishments in 33 developing countries, they find that countries exhibiting greater time-series volatility in productivity are also characterized by greater cross-sectional dispersion in productivity.

What do our findings imply? Assume that we calibrate a model without idiosyncratic shocks and endogenous exit to fit data generated from a model with shocks and endogenous entry. If there are idiosyncratic shocks and endogenous exit, some plants will actually be having negative profits because they have the option to exit. If we try to fit the model without shocks to data generated from a model with shocks and endogenous entry, we will underestimate fixed operational cost to justify the existence of such low productivity plants. By underestimating operational cost, the model without shocks overestimates the expected value of entering plants. As a result, in order to keep the entry level the same, entry cost must be higher in the model without shocks. Hence, given that distortions on TFP are proportional to the value of plants, a calibrated model without shocks and endogenous entry will overestimate policy distortions on TFP if the data reflects shocks and endogenous entry decisions.

There is an important and growing body of literature that analyzes the impact that policy distortions have on TFP in models with idiosyncratic shocks. A closely related paper to ours is Fattal (2011). He uses a calibrated model with idiosyncratic shocks to show that misallocation carries big losses in welfare when the transitional dynamics is taken into account. Buera et al. (2011) also use firm dynamics and shocks at the plant level. Unlike us, they consider policy
configurations that are individual-specific. They show that misallocation in less
developed countries may be due to well-intended policies that were initially choose
to subsidize productive entrepreneurs to relax their credit constraints.

The paper is organized in the following manner. We start out by describing
the economy in section 2. In section 3 we characterize the steady state. Section 4
shows the characterization of policy distortions on TFP with and without shocks
and why a model without shocks and endogenous exit will overestimate policy
distortions on TFP. Section 5 concludes.

2 The Economy

We introduce distortions that affect operational profits, plants entry and delay-
exit decisions in a version of Hopenhayn (1992) and Hopenhayn and Rogerson
(1993) stochastic models of plant level heterogeneity, where plants productivity
evolves as in Luttmer (2007) according to a standard Brownian motion.

2.1 Households

There is a continuum with measure one of identical households who consume, re-
ceive the plant profits and a government transfer. The representative household’s
utility function is equal to

$$\max \int e^{-\rho t} u(c_t) dt,$$

(1)

where $c_t$ is consumption. We assume that $0 < \rho < 1$ and that $u$ is continu-
ously differentiable, strictly concave, and monotonically increasing. We assume
that the household has an endowment of one unit of time, which is inelastically
supplied. Therefore, household’s budget constraint is equal to

$$c_t = w_t + \pi_t + T_{h,t},$$

where $w_t$, $\pi_t$ and $T_{h,t}$ are labor income, plants’ profits and a lump sum transfer
from the government, respectively.
2.2 Incumbent plants

There is a continuum with measure $M$ of heterogeneous, infinitely lived plants who use labor, $n$ to produce the consumption good according to

$$y(s, n) = s^{1-\gamma}n^\gamma,$$

where $1 < \gamma < 0$. Therefore, the only difference across plants is the level of productivity, $s$. We introduce distortions in a very stylized way. Our choice for the model economy’s distortion function is

$$T = \tau y - T_f,$$

where $y$ is the plants’s output. This choice is based on: (i) we consider that low productivity plants receive subsidies and high productivity plants pay taxes; (ii) policy distortion can be implemented as a combination of a positive output tax rate, $\tau$ paid with a lump sum subsidy that each plant receives, $T_f$.

We allow $s$ to vary across plants and over time. Hence, we assume that the value of $s^{(1-\gamma)}$ for a given plant evolves following a Brownian motion

$$ds^{1-\gamma} = -\mu s^{1-\gamma}dt + \sigma s^{1-\gamma}dz,$$  \hspace{1cm} (2)

where $\mu$ is the drift, $\sigma$ is the standard deviation of the process and $dz$ is the increment of a Wiener process. The drift and the standard deviation satisfy $\frac{1}{2}\sigma^2 < \mu < 2\sigma^2$ in order to have a well-behaved problem.

An incumbent plant with productivity $s$ solves

$$\pi = \max_n (1 - \tau) s^{1-\gamma}n^\gamma - wn + T_f.$$

Plant’s optimal labor demand is

$$n(s|w, T) = s \left( (1 - \tau) \frac{\gamma}{w} \right)^{\frac{1}{1-\gamma}},$$  \hspace{1cm} (3)

and its operational profits are equal to

$$\pi(s|w, T) = s \left( \frac{1 - \tau}{w^\gamma} (\gamma^\gamma - \gamma) \right)^{\frac{1}{1-\gamma}} + T_f.$$  \hspace{1cm} (4)

We also assume that there is a fixed cost of operation, $c_f$, and an entry cost, $c_{entry}$, measured in consumption goods units. If the plant wants to remain
in the economy it must pay the fixed cost. Under these assumptions, plants may find optimal exiting the economy and creating a new plant in every period, \( c_{\text{entry}} \frac{2\sigma^2 \rho}{\mu^2} > c_f \).

The firm chooses to remain in the economy by solving

\[
W(s|w, T) = \max_{\{\text{stay, exit}\}} \{ \pi(s|w, T) - c_f + EW(s + ds|w, T), 0 \}
\]

\[
st : ds^{1-\gamma} = -\mu s^{1-\gamma} dt + \sigma s^{1-\gamma} dz.
\]

Both fixed operational cost and government subsidy imply a minimum plant productivity level, if and only if \( c_f - T_f > 0 \), which we assume throughout.

**Lemma 1.** Suppose that \( c_{\text{entry}} \frac{2\sigma^2 \rho}{\mu^2} > c_f \). Then the minimum plant productivity level, \( s^* \), and the value function of a plant with productivity \( s \), \( W(s|w, T) \), are given by

\[
s^*(s|w, T) = \left( \frac{w^{\gamma}}{\kappa(\tau)} \right)^{\frac{1}{1-\gamma}} \frac{-\beta + \rho + \mu}{1 - \beta} \left( c_f - T_f \right)
\]

and

\[
W(s|w, T) = \frac{(c_f - T_f)}{\rho} \left( \left( \frac{s}{s^*} \right)^{\beta - 1} - 1 \right) + \frac{s}{\rho + \mu} \left( \frac{\kappa(\tau)}{w^{\gamma}} \right)^{\frac{1}{1-\gamma}},
\]

where

\[
\beta = \frac{1}{2} + \frac{\mu}{\sigma^2} - \sqrt{\left( \frac{\mu}{\sigma^2} + \frac{1}{2} \right)^2 + \frac{2\rho}{\sigma^2}} < 0,
\]

and \( \kappa(\tau) = (1 - \tau) (\gamma^\gamma - \gamma) \).

**Proof** See appendix A.1.

Equation (5) highlights the distorting role government subsidies play in this economy, even when given as a lump sum. An increase in the government subsidy increases the option value of remaining in the economy. Therefore, higher subsidies imply a lower minimum productivity level.
2.3 Entering plants

Finally, we assume that potential entering plants make their entry decision taking the productivity distribution, \( G(s) \), and distortions \( T \) as given. As Restuccia and Rogerson (2008) we assume that the potential entrant will optimally decide whether to engage in production after observing their realized draw \( s \). Therefore, the expected present discount value of entry is equal to

\[
\int_{s^* (s|w, T)}^{\infty} W(s|w, T) dG(s) ds - c_{entry} = 0. \tag{8}
\]

2.4 Government

Finally, the government budget constraint satisfies

\[
\int_{s^*}^{\infty} T f(s|s^*) ds = \int_{s^*}^{\infty} s \left( (1 - \tau) \frac{\gamma}{w} \right)^{\gamma \over \gamma - 1} ds - T_f = \frac{T_h}{M}, \tag{9}
\]

where \( M \) is the mass of plants in the economy, and \( f(s|s^*) \) is the plants’ productivity distribution that will be characterized later.

3 Steady state

Policy distortions affect plants operational profits, entry and delay-exit decisions. Therefore the stationary distribution of plants productivity, obtained by using forward Kolmogorov equations subject to boundary conditions determined by the optimal delay-exit decisions, is endogenous and depends on policy distortions.

Associated with the Brownian process, the measure of plant follows the following Kolmogorov forward equation

\[
\frac{\partial f(s, t)}{\partial t} = \mu \frac{\partial f(s, t)}{\partial s} + \frac{\sigma^2}{2} \frac{\partial^2 f(s, t)}{\partial s^2},
\]

with boundary condition \( f(s^*) = 0 \ \forall t \). Moreover, as Luttmer (2007) we assume that the potential entering firm imitate existing plants \( dG(s) = \epsilon f(s) \). Therefore,

\[
\mu \frac{\partial f(s|s^*)}{\partial s} + \frac{\sigma^2}{2} \frac{\partial^2 f(s|s^*)}{\partial s^2} + \epsilon f(s|s^*) = 0 \tag{10}
\]
determines the distribution of new entrants
\[
dG(s|s^*) = \frac{1}{2} \frac{\mu^2}{\sigma^2} f(s|s^*),
\]
(11)
where
\[
f(s|s^*) = \left(\frac{\mu}{\sigma^2}\right)^2 (s - s^*) e^{-\mu/(\sigma^2 (s - s^*))}.
\]
(12)

It is now possible to define a steady state equilibrium in this economy.

**Definition 1.** A steady state equilibrium for this economy is a wage rate, \(w\), an aggregate distribution of plants, \(f(s|s^*)\), a mass of firms \(M\), value functions \(W(s|w, T), \pi(s|w, T)\), policy functions \(n(s|w, T), s^*(s|w, T)\), an aggregate consumption \(C\) and policy distortions \(T\) and \(T_h\) such that:

a) Given wages \(w\), and distortions, \(T\), \(\pi(s|w, T)\) (equation 4) solves the incumbent plants’ problem and \(n(s|w, T)\) (equation 3) is the optimal employment level.

b) \(W(s|w, T)\) is the value of a plant with productivity \(s\) (equation 6), and the minimum productivity plant level is given by solving the delay-exit problem (equation 5).

c) The productivity distribution solves the forward Kolmogorov equation (equation 12).

d) There is free entry of plants.
\[
\int_{s^*}^{\infty} W(s|w, T) \frac{1}{2} \frac{\mu^2}{\sigma^2} f(s|s^*) ds - c_{entry} = 0.
\]
(13)

d) Government satisfies the budget constraint (equation 9).

e) Market of goods and market of labor clear
\[
\frac{C}{M} + c_{entry} + c_f = \int_{s^*}^{\infty} s^{1-\gamma} n^\gamma f(s|s^*) ds
\]
(14)
\[
M \int_{s^*}^{\infty} n f(s|s^*) ds = 1.
\]
(15)

\(\text{See Lemma 2 in Luttmer (2007).}\)
For characterizing the steady state we use equation (13) and replacing the value function, we find that the wage is given by

\[ w^\gamma \kappa(\tau) = \left( \frac{s^* + 2\sigma^2}{\mu} \right)^{\frac{1}{2}} \mu^2 \left( \frac{1}{2\sigma^2} \right)^{\frac{1}{2}} \left( \int_{s^*}^{\infty} \left( \frac{s}{s^*} \right)^{\beta} f(s|s^*)ds - 1 \right) \]

where \( \kappa(\tau) = (1 - \tau)(\gamma^\gamma - \gamma) \). We can combine this expression with equation (6) in order to get

\[ s^* = \left( \frac{1}{\left( s^* + 2\sigma^2 \right) \frac{1}{2}} \mu^2 \left( \frac{1}{2\sigma^2} \right)^{\frac{1}{2}} \int_{s^*}^{\infty} \left( \frac{s}{s^*} \right)^{\beta} f(s|s^*)ds - 1 \right)^{\frac{1}{1 - \beta}} \]

The expression for \( M \) follows directly from the expression of labor market clearing

\[ M = \left( \frac{\beta + 1}{\sigma^2} \right) \frac{1}{(s^* + 2\sigma^2)} \left( \frac{1}{\mu} \right) \]

**Proposition 1.** Suppose that \( \frac{1}{2} \sigma^2 < \mu < 2\sigma^2 \). Then the steady state equilibrium associated with the policy distortions, \( T \), exists and it is unique.

**Proof** See appendix A.2.

In a model with idiosyncratic shocks, the values of the marginal plant, \( s^* \), depends on \( c_f \) and \( c_{\text{entry}} \) in a similar way than in a model without shocks: higher operational (entry) cost are positively (negatively) correlated with the minimum productivity plant level.

More interesting is the relationship between \( s^* \) and \( \sigma^2 \), \( \mu \) and \( \rho \). In appendix A.3 we show that if \( (\beta + 1) < \mu/2\sigma^2 \) and \( \left( \frac{\rho c_{\text{entry}}}{c_f - T_f} \right) \) is higher than a lower bound, \( s^* \) is decreasing in \( \sigma^2 \) and increasing in \( \mu \) and \( \rho \). The logic is quite simple. The option value is increasing in \( \sigma^2 \) and decreasing in \( \mu \) and \( \rho \). If the option value increases there will be low productivity plants that are active (waiting for better days). Table 1 summarizes these findings and Figure 3 provides a numerical illustration. Finally, in appendix A.4 we show that wages and the productivity of the marginal plant are positively correlated if the elasticities of \( w \) respect to...
Figure 1: Productivity cut-off, $s^*$, and wages, $w$, as a function of $\sigma^2$ and $\mu$. Benchmark economy: $\rho = 0.05$, $c_{entry} = 20$, $c_f = 3$, $\gamma = 2/3$, $T_f = 1$ and $\tau = 0.2$. 
Table 1: Changes on $s^*$

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$\mu$ and $\rho$ are higher than $\frac{\mu}{\mu + \rho}$, and the elasticity of $w$ respect to $\sigma^2$ is higher than a lower bound.

An interesting property of the model is that it rationalizes the empirical findings of Collard–Wexler et al. (2011). They find that and cross-sectional dispersion in productivity and time-series volatility of productivity are positively correlated. Moreover, time-series volatility of productivity is measured in the model by the standard deviation of the brownian motion. For measuring cross-sectional dispersion in productivity, let us define the first two moments of the plant productivity distribution.

**Definition 2.** Productivity per plant, $E[s]$, and the variance of the plant productivity, $Var[s]$, in this economy are equal to:

$$E[s] = \int_{s^*}^{\infty} s f(s|s^*)ds = s^* (\tau, T_f) + \frac{2\sigma^2}{\mu}$$  \hspace{1cm} (18)

and

$$Var[s] = \int_{s^*}^{\infty} (s - s^*)^2 f(s|s^*)ds = \frac{2\sigma^2}{\mu} \left( s^* (\tau, T_f) - 1 + \frac{2\sigma^2}{\mu} \right) = \frac{2\sigma^2}{\mu} (E[s] - 1).$$  \hspace{1cm} (19)

We can measure cross-sectional dispersion in productivity by using

$$\frac{Var[s]}{E[s]} = \frac{2\sigma^2}{\mu} (1 - E[s]^{-1}).$$  \hspace{1cm} (20)

Therefore, if changes in time-series volatility of productivity, $\sigma^2$, reduces the minimum plant productivity level, $s^*$, less than proportionally, TFP and time-series volatility of productivity and cross-sectional dispersion in productivity are always positively correlated (see Figure 3). Next proposition formalize this idea.
Figure 2: Cross-sectional dispersion in productivity, \( \frac{\text{Var}[s]}{E[s]} \), and time-series volatility of productivity, \( \sigma^2 \). Benchmark economy: \( \rho = 0.05, c_{\text{entry}} = 20, c_f = 3, \gamma = 2/3, T_f = 1 \) and \( \tau = 0.2 \).

**Proposition 2.** Suppose that \( \| \frac{\partial s^*}{\partial \sigma^2} \| < 1 \). Then TFP and time-series volatility of productivity and cross-sectional dispersion in productivity are always positively correlated.

**Proof** Given that \( \frac{\sigma^2}{2} < \mu < 2\sigma^2 \), if \( \| \frac{\partial s^*}{\partial \sigma^2} \| < 1 \), then \( \frac{\partial E[s]}{\partial \sigma^2} \) and \( \frac{\partial}{\partial \sigma^2} \frac{\text{Var}[s]}{E[s]} > 0 \).

### 4 Idiosyncratic shocks and TFP distortions

What are the implications of idiosyncratic shocks in terms of TFP distortions? In order to answer this question we develop the same model without shocks, and we assume that potential entrants draw the productivity parameter from distribution \( h(s|\hat{s}) \) which takes the same form as \( f(s|s^*) \) in the previous analysis,
and the cutoff is potentially different
\[
h(s|\hat{s}) = \left(\frac{\mu}{\sigma^2}\right)^2 (s - \hat{s})e^{-\mu/\sigma^2(s-\hat{s})}.
\]

In order to clearly differentiate with the previous scenario, we use the notation of \(\hat{x}\) to describe the endogenous variables. Furthermore, since productivity is constant over time, a firm with productivity \(s\), has the following value function
\[
V(s|w, T) = \frac{s \left(\frac{1-\tau}{w^\gamma} (\gamma^\gamma - \gamma)\right)^{\frac{1}{1-\gamma}} - cf + Tf}{\rho \lambda},
\]
where \(\lambda\) is the usual exogenous death rate at which firms exit the economy.

As before, the value function determines the minimum productivity size, \(\hat{s}\), as
\[
\hat{s} = \frac{cf - Tf}{(\frac{1-\tau}{w^\gamma} (\gamma^\gamma - \gamma))^{\frac{1}{1-\gamma}}}.
\]
Finally, the free entry condition in this setting is given by
\[
\int_{\hat{s}}^{\infty} V(s|w, T)h(s|\hat{s})ds = c_{\text{entry}}.
\]

In order to have the same entry in both models, we set \(\frac{1}{\lambda} = \frac{1}{2} \frac{\mu^2}{\sigma^2}\). Solving the steady state for this economy we find that
\[
\hat{s} = \frac{(\hat{s} + \frac{2\sigma^2}{\mu})}{\frac{1}{\lambda}} \left(\frac{\rho c_{\text{entry}}}{cf - Tf} + \frac{1}{\lambda}\right)^{\frac{1}{\gamma}}.
\]
\[
\hat{w} = \left(\kappa(\tau) \left(\frac{\hat{s}}{cf - Tf}\right)^{\frac{1-\gamma}{\gamma}}\right)^{\frac{1}{\gamma}}.
\]

We are interested in the insights we obtain comparing both models in terms of distortions. We use two different measures of productivity: i) productivity per plant, \(E[s]\), and ii) measured TFP. To compute measured TFP we use labor, \(N = 1\), and capital
\[
K = M\delta^{-1} \left(\frac{1}{2} \frac{\mu^2}{\sigma^2}c_{\text{entry}} + cf\right),
\]
which is obtained by using the value of investment in entry and operational costs and a constant depreciation rate, \(\delta\), all of them exogenous, paid by entrants and incumbents, respectively.
Definition 3. Measured TFP in this economy is equal to

\[
\frac{M \left( \int_{s^*}^{\infty} s^{1-\gamma} n^\gamma f(s|s^*)ds \right)}{N^\gamma \left( \frac{M}{\delta} \left( \frac{1}{2} \mu^2 c_{\text{entry}} + c_f \right) \right)^{1-\gamma}} = \left( \frac{s^* (\tau, T_f) + \frac{2\sigma^2}{\mu}}{\frac{1}{\delta} \left( \frac{1}{2} \mu^2 c_{\text{entry}} + c_f \right)} \right)^{1-\gamma}.
\]

Next proposition shows that when idiosyncratic shocks exist at plant level, productivity per firm and TFP are lower than with constant productivity.

**Proposition 3.** When idiosyncratic shocks exist, the marginal plant productivity will be lower than the one found in the same model without shocks and endogenous exit. That is \( s^* < \hat{s} \).

**Proof** See appendix A.5.

Proposition 3 shows a well known property of option theory. Under idiosyncratic shocks, incumbent and entering plants decide to remain in the industry when the option value is non-negative. Therefore, there plants with negative operational profits that delay their decision to exit the market exist. This result has a direct implication on the measures of productivity. The marginal plant productivity is lower than the one found in the same model without idiosyncratic shocks. Next corollary shows that this reduces TFP for any policy distortion.

**Corollary 1.** When idiosyncratic shocks exists, TFP and productivity per plant will be lower than the one found in the same model without shocks and endogenous exit. That is, \( TFP < \hat{TPF} \).

Proposition 3 and Corollary 1 tell us that for a given set of parameters, idiosyncratic shocks and endogenous exit reduces productivity per firm and aggregate TFP.
However, parameters are obtained by calibrating the model to match an observed TFP, wage and productivity per plant distribution. Therefore, assume that we calibrate a model without idiosyncratic shocks and endogenous exit that reproduces the TFP, the wage and the productivity distribution generated by an economy with idiosyncratic shocks and endogenous exit. We require that $\text{TFP} = TFP, s^* = \hat{s}, w = \hat{w}$ and $f(s|s^*) = h(s|\hat{s})$. Following Proposition shows that to match the same data we need to underestimate the fixed operational cost, $\hat{c}_f < c_f$ and to overestimate the fixed entry cost, $\hat{c}_{\text{entry}} > c_{\text{entry}}$.

**Proposition 4.** Calibrated operational costs are always below actual operational costs, and the gap between the two is equal to

$$\text{Gap}_{c_f - \hat{c}_f} = \left(1 + \frac{\mu}{\rho}\right) \left(1 - \beta \right) \left(c_f - T_f\right) > 0.$$ 

**Proof** See appendix A.6.

If there are idiosyncratic shocks and endogenous exit in the economy, some plants will actually be making negative profits because they have the option to exit. Proposition 4 shows that if we calibrate a model without idiosyncratic shocks to fit observed TFP, wages and productivity per plant distribution, this seemingly low productive plants will exist only if the fixed operational cost is underestimated. Moreover, given that

$$\frac{\partial \text{Gap}_{c_f - \hat{c}_f}}{\partial \beta} = \left(\frac{\mu}{\rho}\right) \left(1 - \beta \right)^2 \left(c_f - T_f\right) > 0,$$

the gap increases when the value option of the marginal plant (the parameter $\beta$ increases). The logic is quite simple: If the option value increases, plants remain in the economy (waiting for better days) with higher negative profits.

$$\frac{\partial \text{Gap}_{c_f - \hat{c}_f}}{\partial \sigma^2} = \frac{\partial \text{Gap}_{c_f - \hat{c}_f}}{\partial \beta} \frac{\partial \beta}{\partial \sigma^2} > 0.$$ 

Therefore if shocks are very persistent (a low $\sigma^2$), the gap should be small. The intuition behind this result is that a small standard deviation implies less option
value for firms making negative profits, so they choose to exit sooner, and hence the underestimation becomes smaller.

However when $\rho$ and/or $\mu$ increase, the option value decreases, and the gap decreases. Formally

$$\frac{\partial \text{Gap}_{c_f-\hat{c}_f}}{\partial \rho} = \frac{\partial \text{Gap}_{c_f-\hat{c}_f}}{\partial \beta} \frac{\partial \beta}{\partial \rho} + \frac{\partial \text{Gap}_{c_f-\hat{c}_f}}{\partial \rho} < 0$$

$$\frac{\partial \text{Gap}_{c_f-\hat{c}_f}}{\partial \mu} = \frac{\partial \text{Gap}_{c_f-\hat{c}_f}}{\partial \beta} \frac{\partial \beta}{\partial \mu} + \frac{\partial \text{Gap}_{c_f-\hat{c}_f}}{\partial \mu} = \left( \frac{\mu}{\sqrt{\frac{\mu^2}{\sigma^2} + \frac{1}{2}}} \right) \left( \frac{-\beta}{1-\beta} \right) (c_f-T_f) < 0.$$ 

**Corollary 2.** The gap between $c_f$ and $\hat{c}_f$ is increasing in $\sigma^2$ and decreasing in $\mu$ and $\rho$.

When operational cost is underestimated we must overestimate the entry costs. Why? First, because the value of entry is decreasing in the operational cost (see equation 23). Second, because the value of entry is higher in a model with shocks (again the option value argument). As a result, in order to keep the same entry level, entry cost must be higher. Hence, in order to fit the same observed entry level, entry cost must be larger.³

**Proposition 5.** Calibrated fixed entry costs are larger than the actual ones, and the difference is given by

$$\text{Gap}_{c_{\text{entry}}-\hat{c}_{\text{entry}}} = \frac{\mu}{\rho} \left[ c_{\text{entry}} + \frac{\mu}{2\sigma^2} \left( \frac{\hat{c}_f-T_f}{-\beta} \right) \left( 1 - \int_{s^*}^{\infty} \left( \frac{s}{s^*} \right)^{\beta} f(s|s^*)ds \right) \right] > 0$$

**Proof** See appendix A.6.

³We are gratefully to the referee who provide us this argument.
Note that when calibrating lower operational cost and higher fixed cost, the model without idiosyncratic shocks, overestimates the value of the plant (see equation 21). Given that TFP distortions are proportional to the plant’s value, it also overestimates policy distortions on TFP. Formally, relative TFP distortions can be written as

\[
\frac{\partial s^*}{\partial T_f} = \frac{c_{entry}}{\hat{c}_f - T_f} \frac{\rho + \mu}{\rho (1 - \Lambda)}
\]

where

\[
\Lambda = \mu \int_{s^*}^{\infty} \left( \frac{s}{s^*} \right)^\beta \left( \frac{1 - \beta}{s^*} + \frac{1 - \beta}{s} \left( \frac{\mu}{\sigma^2} - \frac{1}{s - s^*} \right) \right) \frac{1}{2} \left( \frac{\mu^2}{\sigma^2} f(s|s^*) ds \right)
\]

The first term of equation (26) shows that distortions are proportional to the value of the plant distorted. A change in distortions is relatively more important in an economy without shocks because the calibrated cost of operation is smaller and the fixed cost is higher than in an economy without shocks. Then plants’ value and distortions will be overestimated. However, there is a second term that shows that the bias in the distortion can be partially broadened. How plants evaluate future in a dynamic model and the shift in the distribution due to a change in the distortion can underestimate the bias. However, next proposition shows that the first effect dominates.

**Proposition 6.** *Calibrated models without idiosyncratic shocks and endogenous exit overestimate policy distortions on TFP*

**Proof** See appendix A.6.

Therefore, assuming that incumbent plants always have non-negative profits and overestimating the plant’s value the relative size of distortions will be overestimated. Hence a calibrated model without idiosyncratic shocks and endogenous exit overestimates the policy distortion on TFP if the data reflects shocks and endogenous entry decisions.
Conclusion

We have analyzed policy distortions at plant level in a tractable stochastic model with analytical solutions. We have compared TFP in our model with TFP without idiosyncratic shocks and endogenous exit in two different ways.

First, we have found that models that do not account for firm dynamics have always larger levels of TFP than models with idiosyncratic shocks and endogenous exit. This result is due to the option of exiting very inefficient plants have in the stochastic case. This choice variable allows plants to remain longer in the market, getting more unproductive, and therefore dropping the level of TFP.

Second, we have shown that if we calibrate a model without idiosyncratic shock and endogenous exit to fit data generated from a model with shocks and endogenous entry, we will always overestimate the effect of policy distortion on TFP.

We assumed that the reallocation of resources was achieved through a policy configuration, that is applied at plant-level that subsidizes low productivity plants and taxes high productivity plants. However, it is well known that idiosyncratic policy distortions affect TFP for all policy configurations since the policy moves plants away from their optimal size.

It is possible to show that similar results can be obtained for more general policy configurations. For example, assume that, at the time of entry, the tax rate is a lottery that once revealed remains fixed for the duration of the time for which the plant is in operation. If we underestimate the future option value of the marginal plants, which exit the economy, we will infer that fixed operating costs are low to justify the existence of the observed exit rate.

Therefore, our results can be extended for any policy configuration that results from lotteries. For example, in Buera et al. (2011) policy initially reallocates capital from unproductive towards productive plants. They assume that policies has inertia and are hard to adjust. In that case, over time, as the productivities of subsidized plants revert to the mean, subsidized plants are not necessarily the most productive, while newly entering, productive plants are taxed. They show that this policy configuration results in the long run equivalent to a lottery: idiosyncratic taxes and subsidies are uncorrelated with the productivities of the
plant distribution.

There are several future extensions of our findings. For instance, we assume that idiosyncratic shocks follow an exogenous brownian motion. However, Aw et al (2008) show that plants invest in their future productivity endogenizing plant dynamics. Moreover, we implicitly assume that there exits a financial sector that allows plants to produce with negative profits. However, Amaral et al (2010) provide evidence that in countries with large informal sector, plants have access to less outside financing. Therefore, considering dynamics more generally would affect the TFP overestimation.

References


A Appendix

A.1 Proof of Lemma 1

We follow Dixit and Pindyck (1994) to solve this exit-delay problem. We know that the firm chooses to stay in the market as long as the firm has a positive option value. Hence,

\[ W(s|w, T) = \max_{\{stay, exit\}} \{ \pi(s|w, T) - cf + EW(s + ds|w, T), 0 \} \]

\[ st : \quad ds^{1-\gamma} = -\mu s^{1-\gamma} dt + \sigma s^{1-\gamma} dz. \]

Using Ito’s calculus, \( W(s|w, T) \) satisfies

\[ \rho W(s|w, T) = \pi(s|w, T) - cf - \mu W'(s|w, T)s + \frac{1}{2} \sigma^2 s^2 W''(s|w, T). \]

The value function that solves the previous expression is given by

\[ W(s|w, T) = Bs^\beta + \frac{s}{\rho + \mu} \left( \frac{\kappa(\tau)}{w^\gamma} \right)^{\frac{1}{1-\gamma}} - \frac{cf - Tf}{\rho}, \]

where \( \kappa(\tau) = (1 - \tau)(\gamma^\gamma - \gamma) \) and \( \beta = \frac{1}{2} + \frac{\mu}{2\sigma^2} - \sqrt{\left( \frac{\mu}{2\sigma^2} + \frac{1}{2} \right)^2 + \frac{2\sigma^2}{4}} < 0 \). Using the smooth-pasting and value matching conditions for \( s^* \), that ensure continuity and differentiability

\[ W(s^*|w, T) = 0 \]
\[ W'(s^*|w, T) = 0 \]

gives

\[ s^*(s|w, T) = \left( \frac{w^\gamma}{\kappa(\tau)} \right)^{\frac{1}{1-\gamma}} - \frac{\beta}{1 - \beta} \left( \frac{cf - Tf}{\rho} \right) \frac{1 + \mu}{\rho} \]

and

\[ W(s|w, T) = \frac{(cf - Tf)}{\rho} \left( \left( \frac{s}{s^*} \right)^{\beta} \frac{1}{1 - \beta} - 1 \right) + s \left( \frac{\kappa(\tau)}{w^\gamma} \right)^{\frac{1}{1-\gamma}} \frac{1}{\rho + \mu}. \]

A.2 Proof of Proposition 1

We must show that this cutoff is unique.

\[ \frac{s^*}{s^* + \frac{2\sigma^2}{\mu}} \left( \frac{1}{2} \frac{\mu^2}{\sigma^2} \left( \frac{1}{1 - \beta} \right) \right) = \frac{\gamma_{entry}}{cf - Tf} - \int_{s^*}^{\infty} \left( \frac{s}{s^*} \right)^{\beta} \left( \frac{1}{1 - \beta} - 1 \right) \frac{1}{\sigma^2} f(s|s^*) ds \]
The left hand side of this expression is 0 when \( s^* \) is 0 and it approaches to 1 as \( s^* \to \infty \). The same limits for the right hand side are

\[
\lim_{s^* \to 0} \frac{\rho_{\text{entry}}}{c_f - T_f} - \int_{s^*}^{\infty} \left( \left( \frac{s}{s^*} \right)^{\beta} \frac{1}{1-\beta} - 1 \right) \frac{1}{2\sigma^2} f(s|s^*) ds \to \frac{1}{\rho_{\text{entry}}/c_f - T_f + \frac{1}{2}\frac{\mu^2}{\sigma^2}} > 0
\]

and

\[
\lim_{s^* \to \infty} \frac{\rho_{\text{entry}}}{c_f - T_f} - \int_{s^*}^{\infty} \left( \left( \frac{s}{s^*} \right)^{\beta} \frac{1}{1-\beta} - 1 \right) \frac{1}{2\sigma^2} f(s|s^*) ds \to \frac{1}{\rho_{\text{entry}}/c_f - T_f + \frac{1}{2}\frac{\mu^2}{\sigma^2}} < 1.
\]

It is trivial that the left hand side of expression (27) is increasing in \( s^* \). The right hand side moves according to the term \( \int_{s^*}^{\infty} \left( \left( \frac{s}{s^*} \right)^{\beta} \frac{1}{1-\beta} - 1 \right) \frac{1}{2\sigma^2} f(s|s^*) ds \). If this term increases, all the right hand side increases as well, and viceversa. In order to proceed the analysis, we multiply the expression by \( 1 - \frac{\mu}{\sigma^2} > 0 \) and take the derivative with respect to \( s^* \)

\[
\frac{d}{ds^*} \int_{s^*}^{\infty} \left( \left( \frac{s}{s^*} \right)^{\beta} - 1 + \beta \right) f(s|s^*) ds = \int_{s^*}^{\infty} \left( -\frac{\beta}{s^*} \left( \frac{s}{s^*} \right) + \left( \left( \frac{s}{s^*} \right)^{\beta} + 1 - \beta \right) \left( -\frac{1}{s-s^*} + \frac{\mu}{\sigma^2} \right) \right) f(s|s^*) ds.
\]

It is clear that the first term dominates the expression for \( s^* \) very close to 0, which makes the sign positive. Since the value has to return to the initial point because the two limits are equal, and the derivative is monotone in \( s^* \), we have that the second term has to be negative. Since the derivative is first increasing and then decreasing, and this path is monotone, we have that the two expressions only intersect once, and hence uniqueness is proved.

### A.3 Derivatives of \( s^* \) respect to \( \rho, \mu, \sigma^2, c_f \) and \( c_{\text{entry}} \)

In the steady state, \( s^* \) satisfies

\[
H(s^*, \rho, \sigma^2, \mu) = \left( s^* + \frac{2\sigma^2}{\mu} \right) \frac{1}{2\sigma^2} \frac{1}{1-\beta} - \frac{\rho_{\text{entry}}}{c_f - T_f} + \frac{1}{2\sigma^2} - \int_{s^*}^{\infty} \left( \left( \frac{s}{s^*} \right)^{\beta} \frac{1}{1-\beta} \right) g(s|s^*) ds
\]

\[
= \lambda \left[ s^* - \frac{s^*}{E \left( \frac{\sigma^2}{\mu} \frac{1}{1-\beta} \right) - \lambda \frac{\rho_{\text{entry}}}{c_f - T_f} + 1 - \int_{s^*}^{\infty} \left( \left( \frac{s}{s^*} \right)^{\beta} \frac{1}{1-\beta} \right) f(\xi) d\xi \right] \right] = 0,
\]

where \( \lambda = \frac{2\sigma^2}{\mu^2}, E = s^* + \frac{2\sigma^2}{\mu} \) and \( \xi = \frac{\mu}{\sigma^2} \). From proposition 1, we know that \( \frac{\partial H}{\partial s^*} > 0 \), because when the two terms intersect the derivative of the first term is necessarily larger.
than that of the second term. We want to calculate

\[
\frac{dH}{d\sigma^2} = \frac{\partial H}{\partial \beta} \frac{\partial \beta}{\partial \sigma^2} + \frac{\partial H}{\partial \sigma^2} \frac{\partial \sigma^2}{\partial \beta} + \frac{\partial H}{\partial \lambda} \frac{\partial \lambda}{\partial \sigma^2} + \frac{\partial H}{\partial \xi} \frac{\partial \xi}{\partial \sigma^2} \\
\frac{dH}{dp} = \frac{\partial H}{\partial \beta} \frac{\partial \beta}{\partial p} + \frac{\partial H}{\partial \sigma^2} \frac{\partial \sigma^2}{\partial p} + \frac{\partial H}{\partial \lambda} \frac{\partial \lambda}{\partial p} + \frac{\partial H}{\partial \xi} \frac{\partial \xi}{\partial p} \\
\frac{dH}{d\rho} = \frac{\partial H}{\partial \beta} \frac{\lambda c_{\text{entry}} s^* (1 - \beta)}{c_f - T_f - \beta E s^*}. 
\]

Therefore, we compute

\[
\frac{\partial H}{\partial \beta} = \frac{s^*}{E_s \beta^2} \left[ 1 - (1 - \beta)^2 \frac{d}{d\beta} \left( \int_{s^*}^{\infty} \left( \frac{s}{s^*} \right)^\beta \frac{1}{1 - \beta} f(\xi) ds \right) \right] \\
\frac{\partial H}{\partial E_s} = \frac{s^*}{E_s^2 \left( \frac{\beta}{1 - \beta} \right)} \\
\frac{\partial H}{\partial \lambda} = \frac{s^*^2 \rho c_{\text{entry}}}{E_s^2 \left( \frac{\beta}{1 - \beta} \right)^2 c_f - T_f} \\
\frac{\partial H}{\partial \xi} = -\frac{s^*^2}{E_s^2 \left( \frac{\beta}{1 - \beta} \right)^2} \frac{d}{d\xi} \left( \int_{s^*}^{\infty} \left( \frac{s}{s^*} \right)^\beta \frac{1}{1 - \beta} f(\xi) ds \right), 
\]

the partial derivatives of \( \beta \)

\[
\frac{\partial \beta}{\partial \rho} = -\frac{1}{\sigma^2 \sqrt{\left( \frac{\mu}{\sigma^2} + \frac{1}{2} \right)^2 + \frac{2\rho}{\sigma^2}}} < 0 \\
\frac{\partial \beta}{\partial \mu} = \frac{1}{\sigma^2} \left( 1 - \frac{\left( \frac{\mu}{\sigma^2} + \frac{1}{2} \right)}{\sqrt{\left( \frac{\mu}{\sigma^2} + \frac{1}{2} \right)^2 + \frac{2\rho}{\sigma^2}}} \right) > 0 \\
\frac{\partial \beta}{\partial \sigma^2} = \frac{\mu}{(\sigma^2)^2} \left( \sqrt{\left( \frac{\mu}{\sigma^2} + \frac{1}{2} \right)^2 + \frac{2\rho}{\sigma^2} + \frac{\mu}{\sigma^2} + \frac{\mu}{\mu^2}} - 1 \right) > 0
\]

and \( \frac{\partial E_s}{\partial \sigma^2} = \frac{2}{\mu} \frac{\partial \lambda}{\partial \sigma^2} = -\frac{2}{\mu^2}, \frac{\partial \lambda}{\partial \sigma^2} = -\frac{\mu}{\sigma^2}, \frac{\partial \xi}{\partial \sigma^2} = -\frac{2\sigma^2}{\mu^2}, \frac{\partial \lambda}{\partial \mu} = -\frac{4\sigma^2}{\mu^3}, \frac{\partial \xi}{\partial \mu} = \frac{1}{\sigma^2}, \) that allow us to write

\[
\frac{dH}{d\sigma^2} = s^* \frac{1}{E_s \beta^2 \sigma^2} \left\{ 1 - (1 - \beta)^2 \frac{d}{d\beta} \left( \int_{s^*}^{\infty} \left( \frac{s}{s^*} \right)^\beta \frac{1}{1 - \beta} f(\xi) ds \right) \right\} \left( 1 - \frac{\left( \frac{\mu}{\sigma^2} + \frac{1}{2} \right)}{\sqrt{\left( \frac{\mu}{\sigma^2} + \frac{1}{2} \right)^2 + \frac{2\rho}{\sigma^2}}} \right) \\
- \frac{1}{2\mu} \frac{(-\beta)(1 - \beta)}{2E_s} + s^*(1 - \beta)^2 \frac{2\sigma^4 \rho c_{\text{entry}}}{\mu^3 (c_f - T_f)} + \frac{d}{d\xi} \left( \int_{s^*}^{\infty} \left( \frac{s}{s^*} \right)^\beta \frac{1}{1 - \beta} f(\xi) ds \right) \right\}
\]
Applying the Laplace transform and the recurrence relation we can prove that if \( \gamma \circ \rho \), we can write

\[
\frac{dH}{d\mu} = \frac{s^*}{E_s} \frac{1}{\beta^2 \sigma^2} \left\{ \left[ 1 - (1 - \beta)^2 \frac{d}{d\beta} \left( \int_{s^*}^{\infty} \left( \frac{s}{s^*} \right)^\beta \frac{1}{1 - \beta} f(\xi) d\xi \right) \right] \left( \frac{\sqrt{\frac{\mu}{\sigma^2} + \frac{1}{2}} + \frac{\rho}{\mu} + \frac{\mu}{\beta^2}}{\sqrt{\left( \frac{\mu}{\sigma^2} + \frac{1}{2} \right)^2 + \frac{2\rho}{\beta^2}} - 1 \right) \right. \\
\left. + \frac{1}{2} \frac{(\beta)(1 - \beta)}{2Es} - s^*(1 - \beta)^2 \frac{d^2}{d\beta^2} \left( \frac{\rho_{\text{entry}}}{c_f - T_f} \right) + \frac{d}{d\zeta} \left( \int_{s^*}^{\infty} \left( s \frac{1}{s^*} (1 - \beta)^2 f(\xi) d\xi \right) \right) \right\}
\]

\[
\frac{dH}{d\rho} = \frac{s^*}{E_s} \frac{1}{\beta^2 \sigma^2} \left\{ \left[ 1 - (1 - \beta)^2 \frac{d}{d\beta} \left( \int_{s^*}^{\infty} \left( \frac{s}{s^*} \right)^\beta \frac{1}{1 - \beta} f(\xi) d\xi \right) \right] \left( \frac{\sqrt{\frac{\mu}{\sigma^2} + \frac{1}{2}} + \frac{\rho}{\mu} + \frac{\mu}{\beta^2}}{\sqrt{\left( \frac{\mu}{\sigma^2} + \frac{1}{2} \right)^2 + \frac{2\rho}{\beta^2}} - 1 \right) \right. \\
\left. + \frac{s^*(1 - \beta)^2}{2\mu^2} \frac{\rho_{\text{entry}}}{c_f - T_f} \right\}.
\]

First, note that \( \frac{dH}{d\sigma^2} \) is increasing and \( \frac{dH}{d\mu} \) and \( \frac{dH}{d\rho} \) are decreasing in \( c_{\text{entry}} \). Second, we can prove that if \((\beta + 1) < \xi/2\) then \( \Phi = \frac{\rho}{s^*} \left( \int_{s^*}^{\infty} \left( \frac{s}{s^*} \right)^\beta \frac{1}{1 - \beta} f(\xi) d\xi \right) \) is positive. Applying the Laplace transform and the recurrence relation

\[
\Gamma(\beta, s^*) = (\beta - 1)\Gamma(\beta - 1, s^*) + s^* \beta - 1 e^{-s^*}
\]

we can write

\[
\Phi = \frac{\xi^2 e^{\xi s^*}}{(1 - \beta) s^*^2} \int_{s^*}^{\infty} \left[ -x^{(\beta+2)} + \frac{2\xi}{\xi + 2s^*} x^{(\beta+1)} - s^*(1 + s^*) x^{\beta} \right] e^{-\xi x} dx >
\]

\[
\frac{\Gamma(\beta + 3, s^*)}{\xi^{(\beta+3)}} + \frac{2\xi}{\xi + 2s^*} \frac{\Gamma(\beta + 2, s^*)}{\xi^{(\beta+2)}} - s^*(1 + s^*) \frac{\Gamma(\beta + 1, s^*)}{\xi^{(\beta+1)}} =
\frac{\Gamma(\beta + 1, s^*)}{\xi^{(\beta+2)}} \left\{ (\beta + 1) \left[ 2\left( \frac{1}{\xi + s^*} - \frac{(\beta + 2)}{\xi} \right) - s^*(1 + s^*) \xi \right] \right\},
\]

where \( \Gamma(a, s^*) = \int_{s^*}^{\infty} x^{a-1} e^{-\xi x} dx \) is the upper incomplete gamma function. Therefore \( \Phi \) is positive if \((\beta + 1)(\beta - 1) \geq s^* \xi [\xi - 2(\beta + 1) - s^*] \), which is true if \((1 + \beta) < \xi/2\). A sufficient condition that guarantees that \( \frac{d\rho_{\text{entry}}}{ds^*} \) is negative, and \( \frac{d\rho_{\text{entry}}}{d\mu} \) and \( \frac{d\rho_{\text{entry}}}{d\rho} \) is positive, is

\[
\left( \frac{\rho_{\text{entry}}}{c_f - T_f} \right) > \frac{E_s \left[ 1 - (1 - \beta)^2 \right] + (-\beta)(1 - \beta)}{s^*(1 - \beta)^2 \frac{\rho_{\text{entry}}}{c_f - T_f}}.
\]

Finally,

\[
\frac{dH}{d\rho_{\text{entry}}} = \frac{s^2}{Es^2} \frac{(1 - \beta)^2}{\beta^2} \left( \frac{\rho_{\text{entry}}}{c_f - T_f} \right)
\]

and

\[
\frac{dH}{d\rho_{\text{entry}}} = - \frac{dH}{d\rho_{\text{entry}}} \left( \frac{\rho_{\text{entry}}}{c_f - T_f} \right).
\]
A.4 Correlation between wages and productivity

Partial derivatives respect $\rho$, $\mu$ and $\sigma^2$ are equal to:

\[
\frac{\partial w}{\partial \rho} = \frac{\partial w}{\partial \beta} \frac{\partial \beta}{\partial \rho} + \frac{\partial w}{\partial s^*} \frac{\partial s^*}{\partial \rho} + \frac{\partial w}{\partial \rho}
= \frac{1 - \gamma}{\gamma} \left( \frac{w}{\beta (1 - \beta) \sigma^2} - \frac{1}{\sqrt{\left( \frac{\mu}{\sigma^2} + \frac{1}{2} \right)^2 + \frac{2\rho}{\sigma^2}}} \right) < 0
+ \frac{\partial s^*}{\partial \rho} \frac{1 - \gamma}{\gamma} \frac{w}{s^*} > 0
+ \frac{1 - \gamma}{\gamma} \frac{w}{\mu} \left( \frac{1}{\rho + \mu} \right) \left( \frac{1}{\sqrt{\left( \frac{\mu}{\sigma^2} + \frac{1}{2} \right)^2 + \frac{2\rho}{\sigma^2}}} \right) > 0
= \frac{1 - \gamma}{\gamma} \frac{w}{\rho} \left[ \frac{\rho}{\beta (1 - \beta) \sigma^2} \frac{1}{\sqrt{\left( \frac{\mu}{\sigma^2} + \frac{1}{2} \right)^2 + \frac{2\rho}{\sigma^2}}} + \frac{\partial s^*}{\partial \rho} \frac{\rho}{s^*} - \frac{\mu}{\rho + \mu} \right] .
\]

\[
\frac{\partial w}{\partial \mu} = \frac{\partial w}{\partial \beta} \frac{\partial \beta}{\partial \mu} + \frac{\partial w}{\partial s^*} \frac{\partial s^*}{\partial \mu} + \frac{\partial w}{\partial \mu}
= \frac{1 - \gamma}{\gamma} \left( \frac{w}{\beta (1 - \beta) \sigma^2} \frac{1}{\sqrt{\left( \frac{\mu}{\sigma^2} + \frac{1}{2} \right)^2 + \frac{2\rho}{\sigma^2}}} \right) > 0
+ \frac{\partial s^*}{\partial \mu} \frac{1 - \gamma}{\gamma} \frac{w}{s^*} > 0
+ \frac{1 - \gamma}{\gamma} \frac{w}{\rho + \mu} < 0
= \frac{1 - \gamma}{\gamma} \frac{w}{\mu} \left[ \frac{\mu}{-\beta (1 - \beta) \sigma^2} \frac{1}{\sqrt{\left( \frac{\mu}{\sigma^2} + \frac{1}{2} \right)^2 + \frac{2\rho}{\sigma^2}}} \right] + \frac{\partial s^*}{\partial \mu} \frac{\mu}{s^*} - \frac{\mu}{\rho + \mu} .
\]
and
\[
\frac{\partial w}{\partial \sigma^2} = \frac{\partial w}{\partial \beta} \frac{\partial \beta}{\partial \sigma^2} + \frac{\partial w}{\partial s^*} \frac{\partial s^*}{\partial \sigma^2} + \frac{\partial w}{\partial \sigma^2} = 0
\]
\[
= \frac{1 - \gamma}{\gamma} \frac{w}{\beta (1 - \beta)} \mu \frac{\mu}{\sigma^2} \left( \frac{\sqrt{\left( \frac{\mu}{\sigma^2} + \frac{1}{2} \right)^2 + \frac{2\rho}{\sigma^2} + \frac{\mu}{\rho} + \left( \frac{\rho}{\mu} \right)^2}}{\sqrt{\left( \frac{\mu}{\sigma^2} + \frac{1}{2} \right)^2 + \frac{2\rho}{\sigma^2}}} - 1 \right) + \frac{\partial s^*}{\partial \sigma^2} \frac{1 - \gamma}{\gamma} \frac{w}{s^*} < 0
\]
\[
= \frac{1 - \gamma}{\gamma} \frac{w}{\sigma^2} \left[ -\frac{\mu}{\beta (1 - \beta)} \frac{1}{\sigma^2} \left( \frac{\sqrt{\left( \frac{\mu}{\sigma^2} + \frac{1}{2} \right)^2 + \frac{2\rho}{\sigma^2} + \frac{\mu}{\rho} + \left( \frac{\rho}{\mu} \right)^2}}{\sqrt{\left( \frac{\mu}{\sigma^2} + \frac{1}{2} \right)^2 + \frac{2\rho}{\sigma^2}}} - 1 \right) + \frac{\partial s^*}{\partial \sigma^2} \frac{\sigma^2}{s^*} \right].
\]

Therefore, if the elasticities of \( w \) respect to \( \mu \) and \( \rho \) are higher than \( \frac{\mu}{\mu + \rho} \), and the elasticity of \( w \) respect to \( \sigma^2 \) is higher than
\[
\frac{2}{\beta (1 - \beta)} \left( \frac{\sqrt{\left( \frac{\mu}{\sigma^2} + \frac{1}{2} \right)^2 + \frac{2\rho}{\sigma^2} + \frac{\mu}{\rho} + \left( \frac{\rho}{\mu} \right)^2}}{\sqrt{\left( \frac{\mu}{\sigma^2} + \frac{1}{2} \right)^2 + \frac{2\rho}{\sigma^2}}} - 1 \right),
\]
then wages and productivity are positively correlated.

### A.5 Proof of Proposition 3

We rewrite equations (16) and (24) in order to have the same term \( \left( \frac{s^* + \frac{2\rho}{\mu}}{\left( \frac{s^* + \frac{2\rho}{\mu} \mu}{\rho} \right) \frac{1}{2} \frac{\mu}{\sigma^2}} \right) \) on the one side, and we compare the remaining parts. It is clear that when a term is written in the following manner \( \frac{x}{(x+K_1)K_2} \), it is increasing in \( x \) for \( K_1, K_2 \) positive. Recall that
\[
\frac{1}{2} \mu^2 = \frac{1}{\lambda}. \text{ Hence,}
\]

\[
\frac{s^*}{(s^* + \frac{2\sigma^2}{\mu}) \frac{1}{2} \sigma^2} = \frac{\rho_{\text{entry}}}{c_f - T_f} - \int_{s^*}^{\infty} \left( \left( \frac{s}{s^*} \right)^\beta \frac{1}{1-\beta} - 1 \right) \frac{1}{2} \frac{\mu^2}{\sigma^2} f(s|s^*) ds = \frac{1}{\rho_{\text{entry}} / (c_f - T_f)} - \int_{s^*}^{\infty} \left( \left( \frac{s}{s^*} \right)^\beta \frac{1}{1-\beta} - 1 \right) \frac{1}{2} \frac{\mu^2}{\sigma^2} f(s|s^*) ds.
\]

\[
\hat{s} = \frac{\rho_{\text{entry}}}{c_f - T_f} + \frac{1}{\lambda}
\]

\[
\hat{w} = \left( \left( s^* \frac{1-\beta}{\rho + \mu c_f - T_f} \right) \left( \frac{1}{\rho + \mu c_f - T_f} \right) \frac{1}{1-\gamma} \right)^{\frac{1}{\gamma}}.
\]

The inequality holds because the denominator of the first expression is larger than that of the second expression \(\blacksquare\)

### A.6 Proof of Proposition 4, 5 and 6

In this appendix, we show that the model without shocks and endogenous exit underestimates the fixed cost of production (Proposition 4) and overestimates the fixed entry cost (Proposition 5). Second, we show that the distortion is always overestimated (Proposition 6).

The main variables are:

\[
s^* = \frac{\rho_{\text{entry}}}{c_f - T_f} - \int_{s^*}^{\infty} \left( \left( \frac{s}{s^*} \right)^\beta \frac{1}{1-\beta} - 1 \right) \frac{1}{2} \frac{\mu^2}{\sigma^2} f(s|s^*) ds,
\]

\[
\hat{s} = \frac{\rho_{\text{entry}}}{c_f - T_f} + \frac{1}{\lambda},
\]

\[
w = \left( \left( s^* \frac{1-\beta}{\rho + \mu c_f - T_f} \right) \left( \frac{1}{\rho + \mu c_f - T_f} \right) \frac{1}{1-\gamma} \right)^{\frac{1}{\gamma}},
\]

\[
\hat{w} = \left( \kappa(\tau) \left( \frac{\hat{s}}{c_f - T_f} \right) \right)^{1-\gamma}.
\]
When we calibrate the parameters, we target TFP, the cutoff and wage to be those observed (in our case, the real ones). This leads the following expressions

\[
\frac{1}{\delta} \left( \frac{1}{2} \frac{\mu^2}{\sigma^2} c_{entry} + \hat{c}_f \right) = \frac{1}{\delta} \left( \frac{1}{2} \frac{\mu^2}{\sigma^2} \hat{c}_{entry} + \hat{c}_f \right)
\]

\[
\frac{\rho \hat{c}_{entry}}{\hat{c}_f - T_f} - \frac{\rho c_{entry}}{c_f - T_f} = \left( \frac{1}{1 - \beta} \right) \left( \frac{\rho c_{entry}}{c_f - T_f} - \int_{s^*}^{\infty} \left( \frac{s}{s^*} \right)^{\beta} \frac{\mu^2}{2 \sigma^2} f(s|s^*) ds + 1 \lambda \right)
\]

(28)

From the calibration we find that \( c_f > \hat{c}_f \) (when we calibrate, we underestimate the fixed cost of producing) and that \( \hat{c}_{entry} > c_{entry} \) (we overestimate the fixed entry cost).

The first result is proven in the following Lemma.

**Lemma A.1.** The calibrated fixed cost is below the actual fixed cost

**Proof** Assume it is not

\[
\frac{1 - \beta}{\beta} \frac{\rho}{\rho + \mu} < 1 \\
(1 - \beta) \rho < -\beta (\rho + \mu)
\]

\[
\frac{\rho}{\mu} + \frac{\mu}{\sigma^2} + \frac{1}{2} < \sqrt{\left( \frac{\mu}{\sigma^2} + \frac{1}{2} \right)^2 + \frac{2\rho}{\sigma^2}}
\]

\[
\left( \frac{\rho}{\mu} \right)^2 + 2 \frac{\rho}{\sigma^2} + \frac{\rho}{\mu} < \frac{2\rho}{\sigma^2}
\]

which cannot hold because \( \frac{\rho}{\mu} \) is positive. ■

Using the last equation that relates the two operational fixed costs, we get that the gap between the two costs is

\[
Gap_{\hat{c}_f - c_f} = \left( \frac{-1 - \frac{\beta \mu}{\rho}}{1 - \beta} \right) (c_f - T_f) < 0.
\]

In order to find that the calibrated fixed entry cost is larger than the actual cost, we use equation (28) and subtract the actual cost from the calibrated one

\[
Gap_{\hat{c}_{entry} - c_{entry}} = \frac{1}{\rho} \left( \mu_{entry} + \left( \frac{c_f - T_f \rho + \mu}{1 - \beta} \right) \left( 1 - \int_{s^*}^{\infty} \left( \frac{s}{s^*} \right)^{\beta} f(s|s^*) ds \right) \right) > 0.
\]
Rewriting of the cutoffs

\[ s^* = \frac{\mu \left( \frac{-\beta}{1-\beta} \right)}{\frac{\rho_{\text{entry}}}{c_{\text{f}} - T_f} - \int_s^\infty \left( \left( \frac{s}{s^*} \right)^\beta \frac{1}{1-\beta} - 1 \right) \frac{1}{2 \sigma^2} f(s|s^*) ds - \frac{1}{2 \sigma^2} \left( \frac{-\beta}{1-\beta} \right)} \]

\[ \hat{s} = \frac{\hat{c}_f - T_f}{\rho_{\text{entry}}} \]

we can take the derivatives of the cutoffs with respect to the distortion, which only affects through \( T_f \)

\[ \frac{\partial s^*}{\partial T_f} = -\mu \left( \frac{-\beta}{1-\beta} \right) \frac{\rho_{\text{entry}}}{(c_{\text{f}} - T_f)^2} \]

\[ -\mu \left( \frac{-\beta}{1-\beta} \right) \left( \frac{\rho_{\text{entry}}}{c_{\text{f}} - T_f} - \int_s^\infty \left( \left( \frac{s}{s^*} \right)^\beta \frac{1}{1-\beta} - 1 \right) \frac{1}{2 \sigma^2} f(s|s^*) ds - \frac{1}{2 \sigma^2} \left( \frac{-\beta}{1-\beta} \right) \right)^2 \]

\[ \frac{\partial \hat{s}}{\partial T_f} = \frac{-\mu}{\rho_{\text{entry}}} \]

and combine both expressions to get

\[ \frac{\partial s^*}{\partial T_f} = \frac{\frac{\rho_{\text{entry}}}{c_{\text{f}} - T_f}}{\rho} + \frac{\mu}{\rho} \int_s^\infty \left( \left( \frac{s}{s^*} \right)^\beta \frac{1}{1-\beta} - 1 \right) \frac{1}{2 \sigma^2} f(s|s^*) ds + \int_s^\infty \left( \left( \frac{s}{s^*} \right)^\beta \frac{1}{1-\beta} - 1 \right) \frac{1}{2 \sigma^2} \partial_t(s|s^*) ds \]

The first term is smaller than one and the rest seem to be larger than one. Yet, we can make use of the ratio \( \frac{\rho_{\text{entry}}}{c_{\text{f}} - T_f} \) from the calibrated part, to get

\[ \frac{\rho_{\text{entry}}}{c_{\text{f}} - T_f} = 1 - \frac{\beta}{1-\beta} \left( -\beta + \int_s^\infty \left( \left( \frac{s}{s^*} \right)^\beta - 1 \right) \frac{1}{2 \sigma^2} f(s|s^*) ds \right) \]

Then, the ratio of the effects can be rewritten as

\[ \frac{\partial s^*}{\partial T_f} = \frac{-\beta}{1-\beta} \left( \frac{\rho_{\text{entry}}}{c_{\text{f}} - T_f} \right) - \beta \left( \frac{\rho_{\text{entry}}}{c_{\text{f}} - T_f} \right)^2 - \rho_{\text{entry}} \int_s^\infty \left( 1 - \left( \frac{s}{s^*} \right)^\beta \right) \frac{1}{2 \sigma^2} f(s|s^*) ds \]

The last term is smaller than one as long as

\[ \frac{\rho_{\text{entry}}}{c_{\text{f}} - T_f} \left( 1 - \int_s^\infty \left( \frac{s}{s^*} \right)^\beta f(s|s^*) ds \right) > \mu \int_s^\infty \left( \left( \frac{s}{s^*} \right)^\beta \left( \frac{-\beta}{s^*} + \frac{\mu}{\sigma^2} - \frac{1}{s - s^*} \right) \right) f(s|s^*) ds. \]
Since we know from Proposition 1 that \( \int_{s^*}^{\infty} \left( \left( \frac{s}{s^*} \right)^{\beta} \left( \frac{\mu}{\sigma^2} - \frac{1}{s-s^*} \right) \right) f(s|s^*) ds < 0 \), whenever \( 1 - (1 - \beta) \int_{s^*}^{\infty} \left( \frac{s}{s^*} \right)^{\beta} f(s|s^*) ds > 0 \), the proof will follow.\(^4\) Thus, all the expression is larger than zero if \( (1 - \beta) \int_{s^*}^{\infty} \left( \frac{s}{s^*} \right)^{\beta} f(s|s^*) ds < 1 \). We use the moment generating function for the function \( s^* e^{-\frac{\mu}{\sigma^2}} (s - s^*) \)

\[
M_s(t) = \int_{s^*}^{\infty} e^{(t - \frac{\mu}{\sigma^2}) s - s^*)} ds,
\]

which gives the moment generating function for our left hand side of the inequality, setting \( \beta = t \), and the inequality we get that the inequality is given by

\[
\frac{e^{\beta s^*} \left( \frac{\mu}{\sigma^2} \right)^2}{s^*^\beta (\beta - \frac{\mu}{\sigma^2})^2} < \frac{1}{1 - \beta}.
\]

We can split the inequality in the following manner

\[
\frac{e^{\beta s^*}}{s^*^\beta} < 1 \quad \text{and} \quad \frac{1 - \beta \left( \frac{\mu}{\sigma^2} \right)^2}{(\beta - \frac{\mu}{\sigma^2})^2} < 1.
\]

Since \( \beta < 0 \), the first inequality is trivial, \( e^{s^*} > s^* \), for all \( s^* \). The second inequality can be rewritten as

\[
\left( \frac{\mu}{\sigma^2} \right)^2 - \beta \left( \frac{\mu}{\sigma^2} \right)^2 < \left( \beta - \frac{\mu}{\sigma^2} \right)^2,
\]

\[
-\beta \left( \frac{\mu}{\sigma^2} \right)^2 < \beta^2 - 2\beta \frac{\mu}{\sigma^2}
\]

\[
\left( \frac{\mu}{\sigma^2} \right)^2 < -\beta + 2 \frac{\mu}{\sigma^2},
\]

which holds, since \( \frac{\mu}{\sigma^2} < 2 \). Hence, we have proven that \( \frac{\partial s^*}{\partial T_f} < \frac{\partial \hat{S}}{\partial T_f} \) for all \( s^* \). ■

\(^4\)To get to this point one should make use of the equality \( s^* = \hat{s} \) and use the definition of the latter.